Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. **TRUE**   False The vertical line test tests whether a curve in the plane is the graph of a function.

   > **Solution:** The vertical line test makes sure there is at most one output to each input.

2. **TRUE**   False While a limit $\lim_{x \to c} f(x)$ does not care what happens exactly at $x = c$ because the limit is concerned only with the behavior of $f(x)$ nearby $x = c$, continuity does care about both and wants them to coincide.

   > **Solution:** Limits only care about what happens near $x = c$ and continuity means that the two are equal.

Show your work and justify your answers. Please circle or box your final answer.

3. True   **FALSE** If a function is not differentiable at $x = c$, then it cannot be continuous there either.

   > **Solution:** A function can be continuous but not differentiable.

4. **TRUE**   False $\sqrt{3}$ can be approximated by using Taylor Polynomials and by Newton's method; however, different functions are needed in each approach.

   > **Solution:** We use the function $\sqrt{x}$ for Taylor Polynomials and $x^2 - 3$ for Newton's method.

5. **TRUE**   False Riemann sums are somewhat cumbersome tools for finding approximations of areas, yet they are absolutely necessary to link antiderivatives to areas.

6. True **FALSE** To calculate the definite integral $\int_{-5}^{5} \sqrt{25 - x^2}\,dx$, we must find an antiderivative of $\sqrt{25 - x^2}$ and use the FTC I to evaluate it at the ends of the interval $[-5, 5]$.

Solution: We can use the area under the curve definition.

7. True **FALSE** $(\ln|x|)' = 1/|x|$ for all $x \neq 0$.

Solution: The derivative is $1/x$.

8. True **FALSE** We can show that $\int_{5}^{\infty} \frac{1}{x^{1.01}}\,dx$ converges in at least three ways: by a brute force calculation using the definition of an improper integral, by representing $\int_{5}^{\infty} \frac{1}{x^{1.01}}\,dx$ as part of $\int_{1}^{\infty} \frac{1}{x^{1.01}}\,dx$ and then using a formula from class for the value of the latter integral, or by comparing it with the more familiar to us integral $\int_{5}^{\infty} \frac{1}{x^1}\,dx$.

Solution: We cannot compare it to $\int_{5}^{\infty} 1/x\,dx$ because that diverges.

9. True **FALSE** For a symmetric distribution centered at 0, we do not have to calculate σ because it will always be 0 or not well-defined.

Solution: The standard deviation will very much not be 0! And also, it may not be defined.

10. True **FALSE** Normal distributions are defined only for positive X; yet, when converted to the standard normal distribution, they may be defined for negative X too.

Solution: Normal distributions are defined for all X.

11. True **FALSE** $P(A \cup B) = P(A) + P(B)$ as long as A and B are independent events in different outcome spaces.

> **Solution:** Addition holds if A and B are non-overlapping.

12. True   **FALSE** For any RV's X and Y, it is true that $E(5X - 7Y) = 5E(X) - 7E(Y)$ and $E(XY) = E(X)E(Y)$.

> **Solution:** The first statement is true but the latter only holds for independent X, Y.